

# Revisiting the $B^0 \rightarrow \pi^0 \pi^0$ decays in the perturbative QCD approach

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(Dated: July 4, 2016)

We recalculate the branching ratio and CP asymmetry for  $\bar{B}^0(B^0) \rightarrow \pi^0 \pi^0$  decays in the Perturbative QCD approach. In this approach, we consider all the possible diagrams including non-factorizable contributions and annihilation contributions, and identity principle is also taken into account. We obtain the branching ratio of  $B^0 \rightarrow \pi^0 \pi^0$  is about  $1.1 \times 10^{-6}$ . Our result is in agreement with the latest measured branching ratio of  $B^0 \rightarrow \pi^0 \pi^0$  by the Belle and HFAG Collaborations. We also predict large direct CP asymmetry and mixing CP asymmetry in  $B^0 \rightarrow \pi^0 \pi^0$  decays, which can be tested by the running LHC-b experiments.

PACS numbers: 13.25.Hw, 11.10.Hi, 12.38.Bx

The detailed study of  $B$  meson decays is a key source of testing the Standard Model(SM), exploring CP violation and in searching of possible new physics beyond the SM. The theoretical studies of  $B$  meson decays have been studied widely in the literature, one of the challenges is that the measured branching ratio [1–3] for the decay of  $B$  meson to neutral pion pairs  $B^0 \rightarrow \pi^0 \pi^0$  is significantly larger than the theoretical predictions obtained in the QCD factorization approach [4–7] or a perturbative QCD approach(PQCD) [8].

The branching ratio of  $B^0 \rightarrow \pi^0 \pi^0$  has been measured, whose data [9] are

$$\left( \begin{array}{l} (1.83 \pm 0.21 \pm 0.13) \times 10^{-6}; (BABAR), \\ (0.90 \pm 0.12 \pm 0.10) \times 10^{-6}; (Belle), \\ (1.17 \pm 0.13) \times 10^{-6}; (HFAG). \end{array} \right). \quad (1)$$

In the last more than 10 years, many theoretical teams have calculated this decays in different approach. Beneke and Neubert made the analysis of  $B^0 \rightarrow \pi^0 \pi^0$  decay based on QCD factorization in 2003 [5]. Recently, Qin Chang [10], Xin Liu [11] and Cong-Feng Qiao [12] *et al.* recalculated this decay model using different method. The next-leading-order (NLO) contributions from the vertex corrections, the quark loops, and the magnetic penguins have also been calculated in the literature [13–16]. By comparing their results, we find the agreement between the theoretical predictions and the experimental data is still not satisfactory, so we revisit the decays of  $B^0 \rightarrow \pi^0 \pi^0$  in this paper. We use the PQCD approach to calculate directly this decays, non-factorizable contributions and annihilation contribution and identity principle are all taken into account.

For the considered  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  decays, the corresponding weak effective Hamiltonian is given by [17].

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ud}^* V_{ub} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] - V_{td}^* V_{tb} \left[ \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\} + \text{H.c.}, \quad (2)$$

where  $C_i(\mu)$  are Wilson coefficients at the renormalization scale  $\mu$  and  $O_i$  are the local four-quark operators

(1) current-current(tree) operators

$$O_1 = (\bar{u}_\alpha u_\alpha)_{V-A} (\bar{d}_\beta b_\beta)_{V-A}, \quad O_2 = (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{d}_\beta u_\beta)_{V-A}; \quad (3)$$

(2) QCD penguin operators

$$\begin{aligned} O_3 &= (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A}, \quad O_4 = (\bar{d}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\ O_5 &= (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A}, \quad O_6 = (\bar{d}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}; \end{aligned} \quad (4)$$

(3) electroweak penguin operators

$$\begin{aligned} O_7 &= \frac{3}{2} (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V+A}, \quad O_8 = \frac{3}{2} (\bar{d}_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A}, \\ O_9 &= \frac{3}{2} (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V-A}, \quad O_{10} = \frac{3}{2} (\bar{d}_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A}. \end{aligned} \quad (5)$$

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Here  $\alpha$  and  $\beta$  are  $SU(3)$  color indices. Then the calculation of decay amplitude is to compute the hadronic matrix elements of the local operators.

In the PQCD approach, the decay amplitude can be written as

$$Amplitude \sim \int d^4k_1 d^4k_2 d^4k_3 Tr[C(t)\Phi_{\bar{B}^0}(k_1)\Phi_{\pi^0}(k_2)\Phi_{\pi^0}(k_3)H(k_1, k_2, k_3, t)]e^{-S(t)}. \quad (6)$$

In our following calculations, the  $B^0$  meson wave function, and the wave functions of pion mesons and relevant distribution amplitudes  $\Phi_{\pi}^{A,P,T}$  are of the same form as those adopted in Refs. [18]. the Gegenbauer moments  $a_i^{\pi}$  and other parameters are adopted from Refs.[18, 19]

$$\begin{aligned} a_1^{\pi} &= 0, & a_2^{\pi} &= 0.25, & a_4^{\pi} &= -0.015, \\ \rho_{\pi} &= m_{\pi}/m_{0\pi}, & \eta_3 &= 0.015, & \omega_3 &= -3.0 \end{aligned} \quad (7)$$

with  $m_{0\pi}$  the chiral mass of the pion.

Fig. 1 shows the lowest order diagrams to be calculated in the PQCD approach for  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  decay. The sum contributions of the non-factorizable diagrams (a) and (b) which come from the operator  $O_2$  are

$$\begin{aligned} \mathcal{M}_a &= \frac{M_B^4}{2} \frac{-1}{\sqrt{2N_c}} 32\pi C_F \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1, b_1) \{ [(x_2 - 2)\Phi_{\pi}^A(x_2)\Phi_{\pi}^A(x_3) \\ &\quad + r_{\pi}(1 - 2x_2)\Phi_{\pi}^T(x_2)\Phi_{\pi}^A(x_3) + r_{\pi}(1 - 2x_2)\Phi_{\pi}^P(x_2)\Phi_{\pi}^A(x_3)]\alpha_s(t_a^1)h_a^1(x_1, x_2, x_3, b_1, b_2) \\ &\quad \exp[-S_B(t_a^1) - S_{\pi}(t_a^1) - S_{\pi}(t_a^1)]C(t_a^1) - 2r_{\pi}\Phi_{\pi}^P(x_2)\Phi_{\pi}^A(x_3)\alpha_s(t_a^2) \\ &\quad h_a^2(x_1, x_2, x_3, b_1, b_2) \exp[-S_B(t_a^2) - S_{\pi}(t_a^2) - S_{\pi}(t_a^2)]C(t_a^2)\}, \end{aligned} \quad (8)$$

where  $C_F = 4/3$  is the group factor of the  $SU(3)_c$  gauge group and  $r_{\pi} = M_{0\pi}/M_B$ , the Sudakov factor  $S_X(t)$  ( $X = \bar{B}^0, \pi^0, \pi^0$ ) can be found in the appendix of Ref.[20]. The functions  $h_a^{1,2}(x_1, x_2, x_3, b_1, b_2)$  come from the Fourier transformation of propagators of virtual quark and gluon. They are defined by

$$\begin{aligned} h_a^j(x_1, x_2, x_3, b_1, b_2) &= \\ &\left\{ \theta(b_1 - b_2)I_0(M_B\sqrt{x_1(1-x_2)}b_2)K_0(M_B\sqrt{x_1(1-x_2)}b_1) \right. \\ &\quad \left. + (b_1 \leftrightarrow b_2) \right\} \times \begin{cases} (K_0(M_B F_{a(j)} b_1), & \text{for } F_{a(j)}^2 > 0 \\ \frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{|F_{a(j)}^2|} b_1), & \text{for } F_{a(j)}^2 < 0 \end{cases}, \end{aligned} \quad (9)$$

where  $F_{a(j)}$ 's are defined by

$$\begin{aligned} F_{a(1)}^2 &= 1 - x_2, \\ F_{a(2)}^2 &= x_1. \end{aligned} \quad (10)$$

The total contribution for the non-factorizable diagrams (c) and (d) is

$$\begin{aligned} \mathcal{M}_c &= \frac{M_B^4}{2} \frac{-1}{\sqrt{2N_c}} 32\pi C_F \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \Phi_B(x_1, b_3) \{ [\Phi_{\pi}^A(x_2)\Phi_{\pi}^A(x_3)(1 - x_1 - x_3) \\ &\quad + r_{\pi}\Phi_{\pi}^P(x_2)\Phi_{\pi}^A(x_3)(1 - x_2) + r_{\pi}\Phi_{\pi}^T(x_2)\Phi_{\pi}^A(x_3)(1 - x_2)]\alpha_s(t_c^1)h_c^1(x_1, x_2, x_3, b_2, b_3) \\ &\quad \exp[-S_B(t_c^1) - S_{\pi}(t_c^1) - S_{\pi}(t_c^1)]C(t_c^1) + [-\Phi_{\pi}^A(x_2)\Phi_{\pi}^A(x_3)(1 + x_3 - x_1 - x_2) - r_{\pi}\Phi_{\pi}^P(x_2)\Phi_{\pi}^A(x_3)(1 - x_2) \\ &\quad + r_{\pi}\Phi_{\pi}^T(x_2)\Phi_{\pi}^A(x_3)(1 - x_2)]\alpha_s(t_c^2)h_c^2(x_1, x_2, x_3, b_2, b_3) \exp[-S_B(t_c^2) - S_{\pi}(t_c^2) - S_{\pi}(t_c^2)]C(t_c^2)\}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} h_c^j(x_1, x_2, x_3, b_2, b_3) &= \\ &\left\{ \theta(b_2 - b_3)I_0(M_B\sqrt{x_1(1-x_2)}b_3)K_0(M_B\sqrt{x_1(1-x_2)}b_2) \right. \\ &\quad \left. + (b_2 \leftrightarrow b_3) \right\} \times \begin{cases} (K_0(M_B F_{c(j)} b_3), & \text{for } F_{c(j)}^2 > 0 \\ \frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{|F_{c(j)}^2|} b_3), & \text{for } F_{c(j)}^2 < 0 \end{cases}, \end{aligned} \quad (12)$$

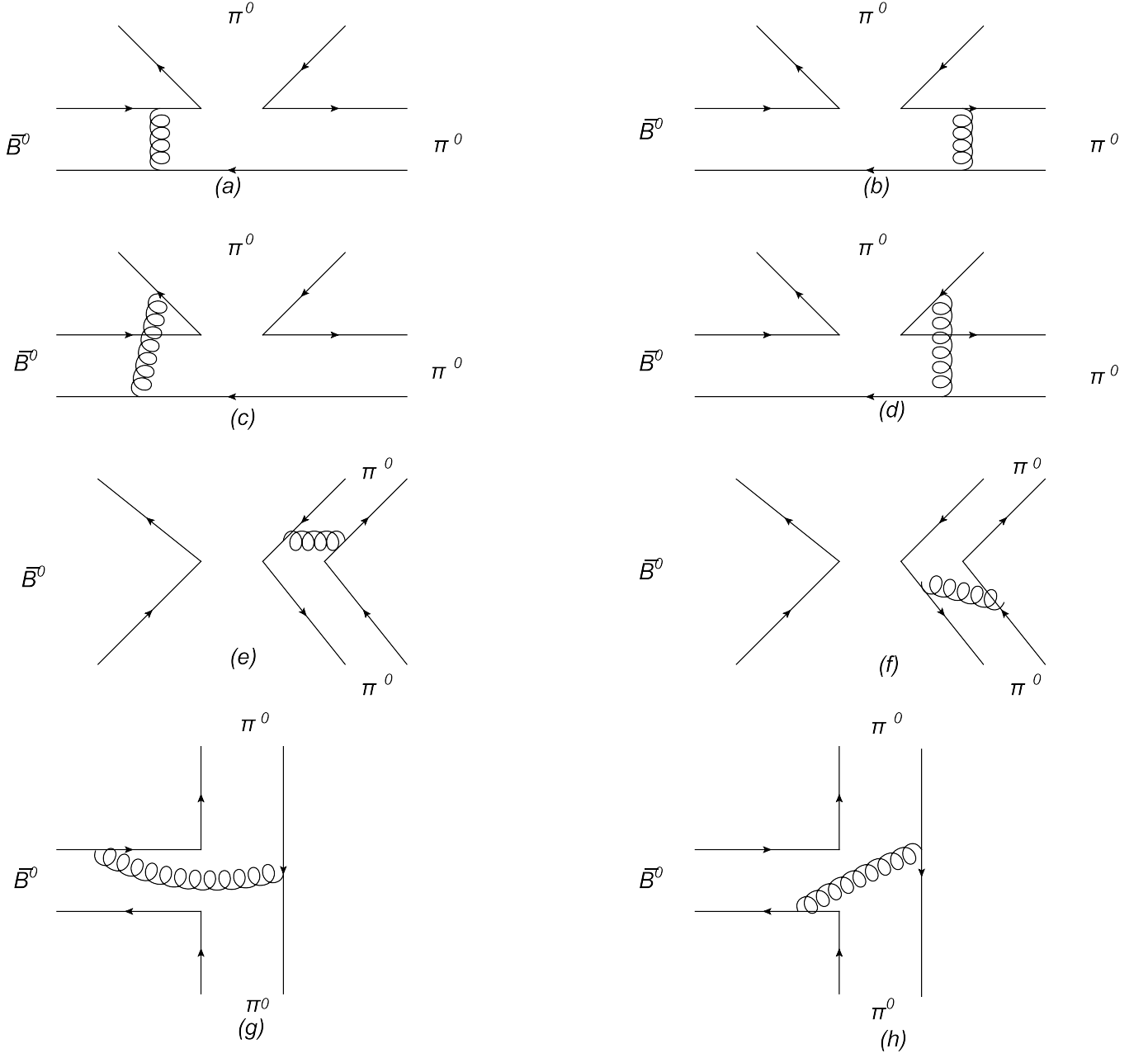


FIG. 1. Typical Feynman diagrams contributing to the  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  decays in the PQCD approach at leading order.

and  $F_{c(j)}$ 's are defined by

$$\begin{aligned} F_{c(1)}^2 &= x_1 + x_2 + x_3 - x_1 x_2 - x_2 x_3 - 1, \\ F_{c(2)}^2 &= x_1 - x_3 - x_1 x_2 + x_2 x_3. \end{aligned} \quad (13)$$

The factorizable annihilation diagrams (e) and (f) which come from the operators  $O_1, O_3, O_4, O_5, O_6, O_7, O_8, O_9, O_{10}$  involve only two light mesons wave functions.  $M_e$  is for  $(V - A)(V - A)$  and  $(V - A)(V + A)$  type operators, and  $M_e^p$  is for

$(1 + \gamma_5)(1 - \gamma_5)$  type operators:

$$\begin{aligned} \mathcal{M}_e = & \frac{M_B^4}{2} 8S\pi C_F \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{ [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_2 - 2r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(1+x_2) \\ & + 2r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_2-1)]\alpha_s(t_e^1)h_e^1(x_2, x_3, b_2, b_3) \exp[-S_\pi(t_e^1) - S_\pi(t_e^1)]C(t_e^1) \\ & + [\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_3 + 2r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(1+x_3) + 2r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(1-x_3)] \\ & \alpha_s(t_e^2)h_e^2(x_2, x_3, b_2, b_3) \exp[-S_\pi(t_e^2) - S_\pi(t_e^2)]C(t_e^2) \}, \end{aligned} \quad (14)$$

$$\begin{aligned} \mathcal{M}_e^P = & \frac{M_B^4}{2} 8S\pi C_F \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{ [-r_\pi \Phi_\pi^P(x_2)\Phi_\pi^A(x_3)x_2 - r_\pi \Phi_\pi^T(x_2)\Phi_\pi^A(x_3)x_2 \\ & - 2r_\pi \Phi_\pi^A(x_2)\Phi_\pi^P(x_3)]\alpha_s(t_e^1)h_e^1(x_2, x_3, b_2, b_3) \exp[-S_\pi(t_e^1) - S_\pi(t_e^1)]C(t_e^1) + [-2r_\pi \Phi_\pi^P(x_2)\Phi_\pi^A(x_3) \\ & - r_\pi \Phi_\pi^A(x_2)\Phi_\pi^P(x_3)x_3 - r_\pi \Phi_\pi^A(x_2)\Phi_\pi^T(x_3)x_3]\alpha_s(t_e^2)h_e^2(x_2, x_3, b_2, b_3) \exp[-S_\pi(t_e^2) - S_\pi(t_e^2)]C(t_e^2) \}, \end{aligned} \quad (15)$$

where  $S = 2$  comes from the requirement of identity principle and

$$\begin{aligned} h_e^1(x_2, x_3, b_2, b_3) = & S_t(x_2)K_0(M_B\sqrt{x_2x_3}b_3) \\ & \times \{ \theta(b_2 - b_3)I_0(M_B\sqrt{x_2}b_2)K_0(M_B\sqrt{x_2}b_3) + (b_2 \leftrightarrow b_3) \} \end{aligned} \quad (16)$$

$$\begin{aligned} h_e^2(x_2, x_3, b_2, b_3) = & S_t(x_3)K_0(M_B\sqrt{x_2x_3}b_2) \\ & \times \{ \theta(b_2 - b_3)I_0(M_B\sqrt{x_3}b_3)K_0(M_B\sqrt{x_3}b_2) + (b_2 \leftrightarrow b_3) \} \end{aligned} \quad (17)$$

The non-factorizable annihilation diagrams ( $g$ ) and ( $h$ ) come from the operators  $O_4, O_6, O_8, O_{10}$ .  $M_g$  is the contribution containing the operator of type  $(V - A)(V - A)$ , and  $M_g^P$  is the contribution containing the operator of type  $(1 + \gamma_5)(1 - \gamma_5)$ .

$$\begin{aligned} \mathcal{M}_g = & \frac{M_B^4}{2} \frac{1}{\sqrt{2N_c}} 32S\pi C_F \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1, b_1) \{ [(x_1 + x_3)\Phi_\pi^A(x_2)\Phi_\pi^A(x_3) \\ & + r_\pi^2(2 + x_1 + x_2 + x_3)\Phi_\pi^P(x_2)\Phi_\pi^P(x_3) - r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_2 - x_1 - x_3) + r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1 + x_3 - x_2) \\ & - r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(2 - x_1 - x_2 - x_3)]\alpha_s(t_g^1)h_g^1(x_1, x_2, x_3, b_1, b_2) \exp[-S_B(t_g^1) - S_\pi(t_g^1) - S_\pi(t_g^1)]C(t_g^1) \\ & + [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_2 + r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(x_1 - x_2 - x_3) - r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_1 - x_3 + x_2) \\ & - r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1 - x_3 + x_2) + r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(x_1 - x_2 - x_3)]\alpha_s(t_g^2) \\ & h_g^2(x_1, x_2, x_3, b_1, b_2) \exp[-S_B(t_g^2) - S_\pi(t_g^2) - S_\pi(t_g^2)]C(t_g^2) \}, \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{M}_g^P = & \frac{M_B^4}{2} \frac{-1}{\sqrt{2N_c}} 32S\pi C_F \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1, b_1) \{ [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_2 \\ & - r_\pi^2(2 + x_1 + x_2 + x_3)\Phi_\pi^P(x_2)\Phi_\pi^P(x_3) - r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_2 - x_1 - x_3) \\ & + r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1 + x_3 - x_2) + r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(x_1 + x_2 + x_3 - 2)]\alpha_s(t_g^1)h_g^1(x_1, x_2, x_3, b_1, b_2) \\ & \exp[-S_B(t_g^1) - S_\pi(t_g^1) - S_\pi(t_g^1)]C(t_g^1) + [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)(x_1 - x_3) - r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(x_1 - x_2 - x_3) \\ & - r_\pi^2 \Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_1 - x_3 + x_2) + r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1 + x_2 - x_3) - r_\pi^2 \Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(x_2 + x_3 - x_1)] \\ & \alpha_s(t_g^2)h_g^2(x_1, x_2, x_3, b_1, b_2) \exp[-S_B(t_g^2) - S_\pi(t_g^2) - S_\pi(t_g^2)]C(t_g^2) \}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} h_g^j(x_1, x_2, x_3, b_1, b_2) = & \left\{ \theta(b_1 - b_2)I_0(M_B\sqrt{x_2x_3}b_1)K_0(M_B\sqrt{x_2x_3}b_2) \right. \\ & \left. + (b_1 \leftrightarrow b_2) \right\} \times \left( \begin{aligned} & (K_0(M_B F_{g(j)} b_1), \quad \text{for } F_{g(j)}^2 > 0) \\ & \frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{|F_{g(j)}^2|} b_1), \quad \text{for } F_{g(j)}^2 < 0 \end{aligned} \right), \end{aligned} \quad (20)$$

and  $F_{g(j)}$ 's are defined by

$$\begin{aligned} F_{g(1)}^2 &= x_1 + x_2 + x_3 - x_1x_2 - x_2x_3, \\ F_{g(2)}^2 &= x_1x_2 - x_2x_3. \end{aligned} \quad (21)$$

The total decay amplitude of  $\bar{B}^0 \rightarrow \pi^0\pi^0$  is then

$$\begin{aligned} \bar{A}(\bar{B}^0 \rightarrow \pi^0\pi^0) &= V_{ud}^*V_{ub}[C_1\mathcal{M}_e f_B + C_2(\mathcal{M}_a + \mathcal{M}_c)] - V_{td}^*V_{tb}[(2C_3 + \frac{5}{3}C_4 + 2C_5 + \frac{2}{3}C_6 + \frac{1}{2}C_7 \\ &\quad + \frac{1}{6}C_8 + \frac{1}{2}C_9 - \frac{1}{3}C_{10})\mathcal{M}_e f_B + (C_6 - \frac{1}{2}C_8)\mathcal{M}_e^P f_B + (2C_4 + \frac{1}{2}C_{10})\mathcal{M}_g + (2C_6 + \frac{1}{2}C_8)\mathcal{M}_g^P] \end{aligned} \quad (22)$$

and the decay width is expressed as

$$\Gamma(\bar{B}^0 \rightarrow \pi^0\pi^0) = \frac{G_F^2 M_B^3}{128\pi} |\bar{A}(\bar{B}^0 \rightarrow \pi^0\pi^0)|^2 \quad (23)$$

The decay amplitude of the charge conjugate channel for  $\bar{B}^0 \rightarrow \pi^0\pi^0$  can be obtained by replacing  $V_{ud}^*V_{ub}$  to  $V_{ud}V_{ub}^*$  and  $V_{td}^*V_{tb}$  to  $V_{td}V_{tb}^*$  in Eq. (22). The decay amplitude of  $\bar{B}^0 \rightarrow \pi^0\pi^0$  in Eq. (22) can be parameterized as

$$\bar{A} = V_{ud}^*V_{ub}T - V_{td}^*V_{tb}P = V_{ud}^*V_{ub}T[1 + ze^{i(-\alpha+\delta)}], \quad (24)$$

where  $z = |V_{td}^*V_{tb}/V_{ud}^*V_{ub}| |P/T|$ , and  $\delta = \arg(P/T)$  is the relative strong phase between tree diagrams T and penguin diagrams P.  $z$  and  $\delta$  can be calculated from PQCD.

Similarly, the decay amplitude for  $B^0 \rightarrow \pi^0\pi^0$  can be parameterized as

$$\mathcal{A} = V_{ub}^*V_{ud}T - V_{tb}^*V_{td}P = V_{ub}^*V_{ud}T[1 + ze^{i(\alpha+\delta)}]. \quad (25)$$

The following parameters have been used in our numerical calculation [1, 2, 21, 22].

$$\begin{aligned} \Lambda_{QCD}^{f=4} &= 0.25 GeV, \quad m_W = 80.41 GeV, \quad m_B = 5.280 GeV, \\ f_\pi &= 0.13 GeV, \quad f_B = 0.19 GeV, \quad m_{0\pi} = 1.4 GeV, \quad \tau_{B^0} = 1.55 \times 10^{-12} s, \\ |V_{ud}^*V_{ub}| &= 0.00346, \quad |V_{td}^*V_{tb}| = 0.00885. \end{aligned} \quad (26)$$

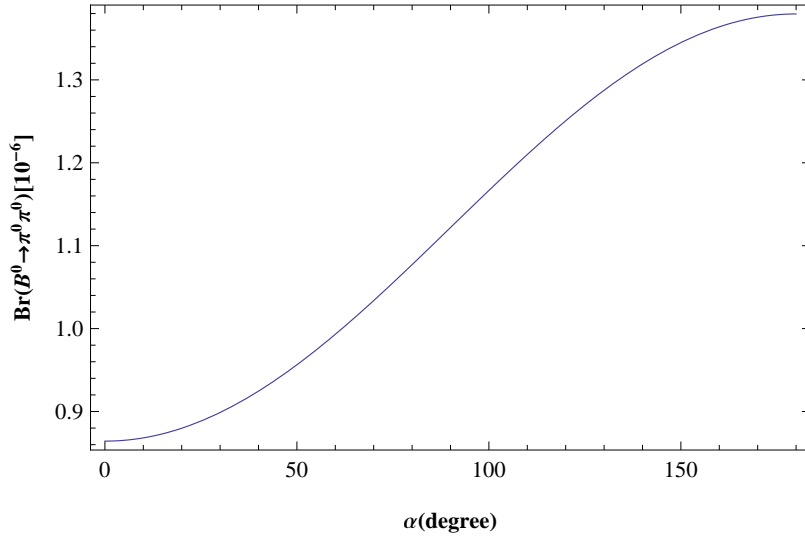


FIG. 2. The averaged branching ratio of  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  decay as a function of CKM angle  $\alpha$ .

We leave the Cabibbo-Kobayashi-Maskawa (CKM) phase angle  $\alpha$  as a free parameter to explore the branching ratio and CP asymmetry. From Eqs. (24) and (25), we get the averaged decay width for  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$

$$\begin{aligned} \Gamma(\bar{B}^0(B^0) \rightarrow \pi^0\pi^0) &= \frac{G_F^2 M_B^3}{128\pi} \left( \frac{|\mathcal{A}|^2}{2} + \frac{|\bar{\mathcal{A}}|^2}{2} \right) \\ &= \frac{G_F^2 M_B^3}{128\pi} |V_{ud}^*V_{ub}T|^2 [1 + 2z \cos(\alpha) \cos(\delta) + z^2]. \end{aligned} \quad (27)$$

Using the above parameters, we get  $z = 0.52$  and  $\delta = 106^\circ$  in PQCD. Equation (27) is a function of CKM angle  $\alpha$ . In Fig. 2, we plot the averaged branching ratio of the decay  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  with respect to the parameter  $\alpha$ . Since the CKM angle  $\alpha$  is constrained as  $\alpha$  around  $85^\circ$  [22].

$$\alpha = (85.4_{-3.8}^{+3.9})^\circ \quad (28)$$

We can arrive from Fig. 2

$$1.08 \times 10^{-6} < Br(\bar{B}^0(B^0) \rightarrow \pi^0\pi^0) < 1.12 \times 10^{-6}, \quad for 80^\circ < \alpha < 90^\circ \quad (29)$$

The number  $z = |V_{td}^* V_{tb}/V_{ud}^* V_{ub}| |P/T| = 0.52$  means that the amplitude of penguin diagrams is about 0.52 times that of tree diagrams, which shows though the tree contribution dominate this decay, the penguin contribution cannot be ignored, i. e., there are large contributions from both tree diagrams and penguin diagrams.

In the literature, there already exist a lot of studies on  $B^0 \rightarrow \pi^0\pi^0$  decay. We give some recent works devoted to the resolution of the challenge:

(a) In Ref. [10], Qin Chang and Junfeng Sun *et al* do a global fit on the spectator scattering and annihilation parameters  $X_H(\rho_H, \phi_H)$ ,  $X_A^i(\rho_A^i, \phi_A^i)$  and  $X_A^f(\rho_A^f, \phi_A^f)$  for the available experimental data for  $B_{u,d} \rightarrow \pi\pi, \pi K$  and  $K\bar{K}$  decays in the QCDF framework. They obtained large  $B^0 \rightarrow \pi^0\pi^0$  branching ratios  $1.67_{-0.30}^{+0.33} \times 10^{-6}$  and  $2.13_{-0.38}^{+0.43} \times 10^{-6}$  for different scenarios.

(b) In Ref. [11], Xin Liu, Hsiang-nan Li and Zhen-Jun Xiao investigate the Glauber-gluon effect on the  $B \rightarrow \pi\pi$  and  $\rho\rho$  decays based on the  $k_T$  factorization theorem, they observed significant modification of  $B^0 \rightarrow \pi^0\pi^0$  branching ratio through a transverse-momentum-dependent(TMD) wave function for the pion with a weak falloff in parton transverse momentum  $k_T$ . They get the branching ratio of the  $B^0 \rightarrow \pi^0\pi^0$   $0.61 \times 10^{-6}$ .

(c) In Ref. [12], Cong-Feng Qiao *et al* give a possible solution to the  $B \rightarrow \pi\pi$  puzzle using the principle of maximum conformality. They found the PQCD prediction is highly sensitive to the choice of the renormalization scale which enter the decay amplitude, they obtained  $Br(B_d \rightarrow \pi^0\pi^0) = (0.98_{-0.31}^{+0.44}) \times 10^{-6}$  by applying the principle of maximum conformality.

(d) In Ref. [23], Ya-Lan Zhang *et al* performed a systematic study for the  $B \rightarrow (\pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0)$  decays in the PQCD factorization approach with the inclusion of all currently known NLO contributions from various sources. They got the NLO PQCD prediction for  $B^0 \rightarrow \pi^0\pi^0$  branching ratio  $Br(B^0 \rightarrow \pi^0\pi^0) = [0.23_{-0.05}^{+0.08}(\omega(b))_{-0.04}^{+0.05}(f_B)_{-0.03}^{+0.04}(a_\pi^2)] \times 10^{-6}$ , it is still much smaller than the measured data.

(e) In Ref. [24], Hai-Yang Cheng, Cheng-Wei Chiang and An-Li Kuo used flavor SU(3) symmetry to analyze the data of charmless  $B$  meson decays to two pseudoscalar mesons ( $PP$ ) and one vector and one pseudoscalar mesons ( $VP$ ). They found the color-suppressed tree amplitude larger than previously known and has a strong phase of  $-70^\circ$  relative to the color favored tree amplitude in the PP sector, this large color-suppressed tree amplitude results in the large  $B^0 \rightarrow \pi^0\pi^0$  branching ratios  $1.43 \pm 0.55 \times 10^{-6}$  and  $1.88 \pm 0.42 \times 10^{-6}$  for different scheme.

There are some works on  $B^0 \rightarrow \pi^0\pi^0$  decay in the framework of PQCD approach before [8, 18, 23]. Ref. [8] is the earliest PQCD calculations for  $B^0 \rightarrow \pi^0\pi^0$  decay at the leading order, Hsiang-nan Li *et al* considered partial NLO contributions in Ref. [18]. Based on the work of Ref. [8, 18], Ya-Lan Zhang *et al* calculated all currently known NLO contributions from various sources in Ref. [23]. All these calculations got a branching ratio which is much smaller than the experimental data. In this work, we recalculate the  $B^0 \rightarrow \pi^0\pi^0$  decay in the framework of PQCD approach. We find the largest contributions come from the factorizable annihilation diagrams (e) and (f) (see Fig. 1), both tree operator ( $O_1$ ) and penguin operators ( $O_3, O_4, O_5, O_6, O_7, O_8, O_9, O_{10}$ ) contribute to this two diagrams. Our result is much larger than that of previous predictions [8, 18, 23], there are two reasons that make the difference. In previous PQCD works [8, 18, 23], first, the contributions of the factorizable annihilation diagrams (e) and (f) come from tree operator  $O_1$  had not been taken into account, the authors only considered the non-factorizable diagrams (a) and (b) (small contributions) for operator  $O_1$ ; second, For  $O_3, O_4, O_9, O_{10}$  operators, previous calculations [8] showed their contributions cancel between diagram (e) and (f), however, we recalculate it and find their contributions cannot be canceled between diagram (e) and (f), as shown in Eqs. (14)(15). If we get rid of the contributions of  $\mathcal{M}_e$  and  $\mathcal{M}_e^P$  terms in Eq. (22), our result is  $Br(\bar{B}^0(B^0) \rightarrow \pi^0\pi^0) \sim 0.26 \times 10^{-6}$ , which is consistent with previous PQCD predictions [8, 18, 23]. In our recalculations, we consider all the possible diagrams's contribution, including non-factorizable contributions and annihilation contributions, and identity principle is also taken into account, we obtain branching ratio of  $B^0 \rightarrow \pi^0\pi^0$  around  $1.1 \times 10^{-6}$ , which is still smaller than BABAR result [9], but it is consistent with the Belle and HFAG results [9]. More experimental and theoretical efforts should be made to resolve the  $B^0 \rightarrow \pi^0\pi^0$  puzzle.

In SM, the CKM phase angle is the origin of CP violation. Using Eqs. (24) and (25), the direct CP violating parameter is

$$\mathcal{A}_{CP}^{\text{dir}} = \frac{|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2} = \frac{2z \sin(\alpha) \sin(\delta)}{1 + 2z \cos(\alpha) \cos(\delta) + z^2} \quad (30)$$

It is approximately proportional to CKM angle  $\sin(\alpha)$ , strong phase  $\sin(\delta)$ , and the relative size  $z$  between penguin contribution and tree contribution. We show the direct CP asymmetry  $\mathcal{A}_{CP}^{\text{dir}}$  in Fig. 3. One can see from the figure that the direct CP asymmetry parameter of  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  can be as large as from  $-80\%$  to  $-78\%$  when  $80^\circ < \alpha < 90^\circ$ . The large direct CP asymmetry is also a result of there are large contributions from both tree diagrams and penguin diagrams in this decays.

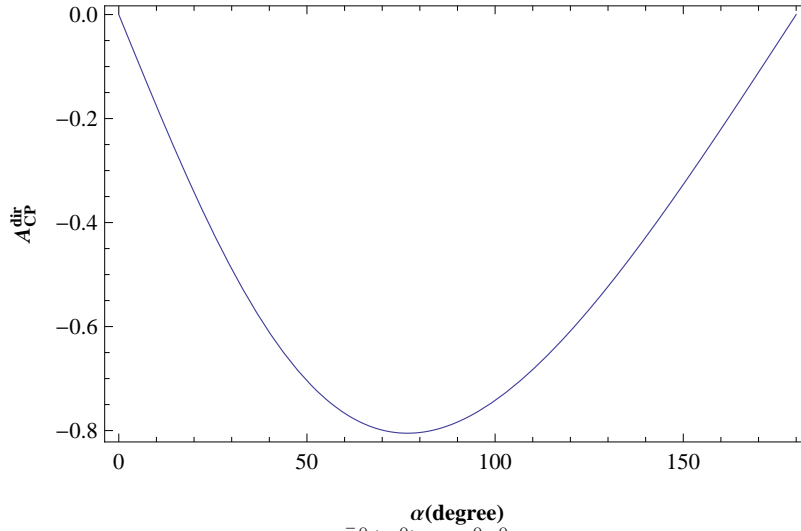


FIG. 3. Direct CP violation parameter of  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  decay as a function of CKM angle  $\alpha$ .

For the neutral  $B^0$  decays, the  $\bar{B}^0 - B^0$  mixing is very complex. Following notations in the previous literature [25], we define the mixing induced CP violation parameter as

$$a_{\epsilon+\epsilon'} = \frac{-2\text{Im}(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \quad (31)$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td} \langle \pi^0 \pi^0 | H_{eff} | \bar{B}^0 \rangle}{V_{tb} V_{td}^* \langle \pi^0 \pi^0 | H_{eff} | B^0 \rangle}. \quad (32)$$

Using equations (24) and (25), we can derive as

$$\lambda_{CP} = e^{2i\alpha} \frac{1 + ze^{i(\delta-\alpha)}}{1 + ze^{i(\delta+\alpha)}} \quad (33)$$

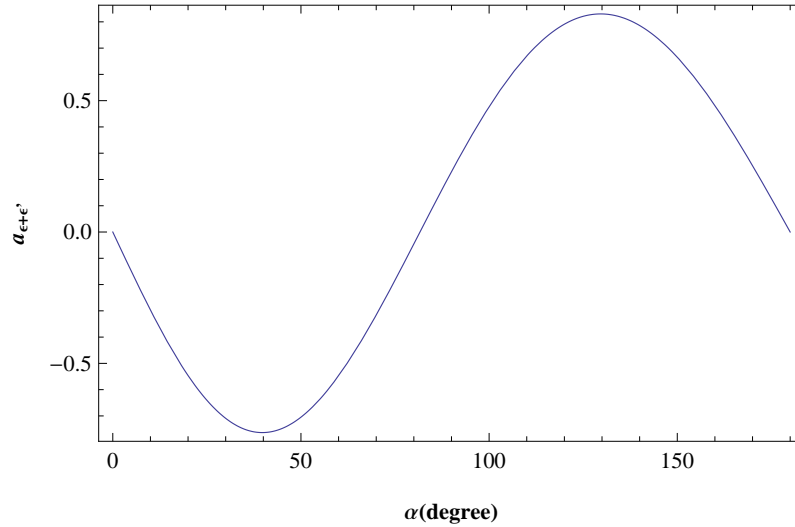


FIG. 4. Mixing CP violation parameter of  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  decay as a function of CKM angle  $\alpha$ .

If  $z$  is a very small number, i. e., the penguin diagram contribution is suppressed comparing with the tree diagram contribution, the mixing induced CP asymmetry parameter  $a_{\epsilon+\epsilon'}$  is proportional to  $-\sin 2\alpha$ , which will be a good place for the CKM angle  $\alpha$  measurement. However as we have already mentioned,  $z$  is not very small. We give the mixing CP asymmetry in Fig. 4, one

can see that  $a_{\epsilon+\epsilon'}$  is not a simple  $-\sin 2\alpha$  behavior because of the so-called penguin pollution. It is close to 9% when the angle near  $85^\circ$ . At present, there are no CP asymmetry measurements in experiment but the possible large CP violation we predict for  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  decays might be observed in the running LHC-b experiments.

In conclusion, we recalculate the branching ratio and CP asymmetry of the decays  $\bar{B}^0(B^0) \rightarrow \pi^0\pi^0$  in PQCD approach. From our calculation, we find the branching ratio of  $B^0 \rightarrow \pi^0\pi^0$  around  $1.1 \times 10^{-6}$ , much larger than that of previous predictions[8], and there are large CP violation in the process, which may be measured in the running LHC-b experiments. The branching ratio we get is still smaller than BABAR result [9], but it is consistent with the latest Belle and HFAG results [9].

We would to thank Dr. Ming-Zhen Zhou and Dr. Wen-Long Sang for valuable discussions. This work is supported by the National Natural Science Foundation of China under Grant No.11047028 and the Fundamental Research Funds of the Central Universities, Grant Number XDJK2012C040.

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